where b is suction velocity at the walls and is a positive constant, ν is the viscosity of the fluid, u is the velocity component along the corner in the x direction, and y and z are the coordinates normal to the plates.

The boundary conditions are

$$u(x,0,z)=0 \quad u(x,y,0)=0 \quad u(x,\infty,\infty)=U$$

where U is the freestream velocity.

Equation (1), when solved with the above boundary conditions, yields the following velocity profile:

$$u/U=(1-e^{-(b/\nu)y})(1-e^{-(b/\nu)z}) \tag{2}$$

The constant velocity contours for $u/U=0.9$ and 0.99 are shown in Fig. 1. It can be observed that when the distance is greater than $6b/\nu$ the interaction is very small. The variation of the wall shearing stress can be calculated by

$$(\tau_{xy})_0=Ub\rho(1-e^{-(b/\nu)z}) \tag{3}$$

where $(\tau_{xy})_0$ is the wall shearing stress in the x-y direction and ρ is the density of the fluid. Equation (3) shows that the wall shearing stress is equal to zero at the intersection line of the plates ($z=0$ and $y=0$) and is equal to ρUb at large distance from the corner. The variation of $(\tau_{xy})_0$ with distance from the corner is shown in Fig. 2.

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Smoke Visualization of Boundary-Layer Transition on a Spinning Axisymmetric Body

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Introduction

BECAUSE of the variety of transitional behavior observed, it is clear that there are a number of possible paths to turbulence.[1] Boundary layers on spinning and nonspinning bodies exhibit two modes of instability and transition which depend upon Reynolds number and spin rate. For axisymmetric bodies without spin, a two-dimensional viscosity-conditioned instability leads to the development of Tollmien-Schlichting waves and their breakdown. On the other hand, an inflectional crossflow instability generates vortices that spiral around the spinning body and break down, possibly after a helical instability. Each of these modes appears to possess distinct topology of nonlinear breakdown and onset of turbulence. The purpose of this Note is to present and discuss smoke flow visualization photographs of these breakdowns and to call attention to a new situation where both modes superimpose simultaneously. It should be noted that, with the possible exception of the Tollmien-Schlichting (T-S) waves, the phenomena, when identifiable from the smoke, are already nonlinear. It is the global and nonintrusive character of the flow visualization that makes these observations possible.

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Experimental Apparatus and Technique

The experiments were conducted in one of the University of Notre Dame's low-turbulence, subsonic smoke wind tunnels. This indraft wind tunnel has 12 antiturbulence screens, followed by a 24:1 contraction in area to the test section which is 610 × 610 mm square and 1828 mm long. The wind tunnel in this configuration can achieve velocities in the range of 5-27 m/s with a turbulence intensity of approximately 0.10% over this range. The activity in the boundary layer was made visible using a single kerosene smoke filament which entered the wind tunnel upstream of the first screen and was positioned to impinge, in a symmetrical fashion, on the sharp nose of the model.

The axisymmetric model consisted of a 3-caliber secant ogive nose, a 2-caliber cylindrical midsection, and a 1-caliber, 7 deg conical boattail. There were discontinuities in the slope of the body surface at the junctions of the nose and midsection, and boattail and midsection. It was polished to a surface finish of 0.254 μm (10 μin.) and anodized black. Still photographs and high-speed movies were taken at an angle of attack of zero, over Reynolds numbers based upon body length R_L from 0.315×10^6 to 1.030×10^6 and spin rates of 0-4500 rpm.

Results

In the nonspinning experiments, two-dimensional (i.e., axisymmetric) Tollmien-Schlichting waves appear sporadically along the body at $R_L = 0.631 \times 10^6$ and appear continuously at all higher Reynolds numbers. At the higher Reynolds numbers, these waves become three-dimensionally unstable as

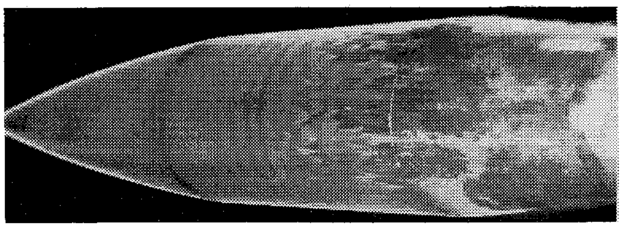

Fig. 1 Smoke photograph of nonspinning axisymmetric body for $\alpha = 0$ deg and $Re_L = 1{,}030{,}000$.

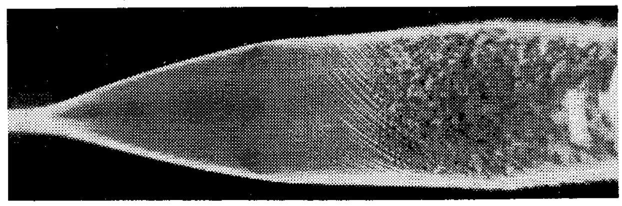

Fig. 2 Typical striations in the smoke resulting from crossflow vortices for $\alpha = 0$ deg, $V/U_\infty = 0.848$ (1250 rpm), and $Re_L = 315{,}000$.

they travel along the body. These three-dimensionally deformed waves break down into the well-known vortex truss pattern which, in turn, breaks down into motions consistent with the concepts of turbulence.[2] The transition phenomenon of Fig. 1 shows many of the characteristics of transition described in the earlier research conducted at Notre Dame by Brown[3] and Knapp and Roache[4] on tangent ogive-nosed models.¶ At the highest Reynolds number studied (1.030×10^6), there is an uninterrupted, although modulated, formation of nearly two-dimensional T-S waves. The formation of these waves is influenced by freestream disturbances. Although the two-dimensional T-S waves appear continuously at essentially the same point on the model, the formation of the vortex trusses is intermittent. The trusses form in groups, periodically appearing anywhere from 0.9 to 1.75 caliber along the body. It is interesting that the tunnel speed at which the transition process occurs entirely on the body is significantly higher than that in previous experiments where the model was longer.[3,4] The present higher velocities yield thinner boundary layers in the transition region, resulting in higher frequencies of the two-dimensional T-S waves and in smaller vortex trusses. A more severe pressure gradient occurs on the model where the secant ogive nose joins the cylindrical midsection than for the tangent ogive-nosed model used by Brown and his associates.

The inflectional mode of transition over the spinning model took a distinctly different form from that of the nonspinning case.[6] The phenomenon was primarily related to the ratio of the peripheral velocity to the freestream velocity V/U_∞ and relatively independent of Reynolds numbers (i.e., it was not significantly affected by changes in Reynolds number for a given V/U_∞). Tests were conducted for a range of V/U_∞ between zero and 1.67. There were no notable changes in the boundary-layer characteristics for V/U_∞ less than 0.4, with the exception of a slight skewness in the tips of the vortex trusses. When vortex trusses were present, this skewness could be seen for V/U_∞ values as low as 0.1. As V/U_∞ increased, striations in the smoke (manifestations of the crossflow instability which produces vortices) appeared at an angle approximately equal to $\tan^{-1}$ of V/U_∞, as shown in Fig. 2. The spinning crossflow instability is closely related to the instabilities found on a swept wing and a rotating disk.[7] Such an inflectional instability is much more unstable than the viscosity-controlled T-S instability and is rather Reynolds number insensitive. The presence of dust on the model or a wetted surface can greatly alter the location and size of the vortices. Otherwise, the vortex development does not appear to be appreciably affected by freestream disturbance. The wavelength of these striations remains approximately constant regardless of spin ratio or Reynolds number. The transition region moves forward with increasing spin rate and the transition process takes place over a shorter distance as

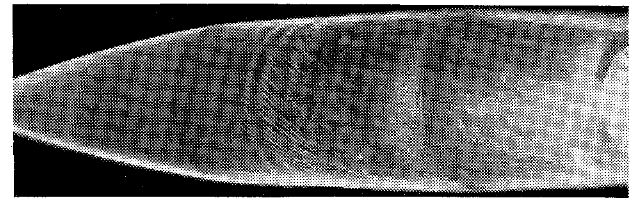

Fig. 3 Simultaneous appearance of Tollmien-Schlichting waves and crossflow vortices for $\alpha = 0$ deg, $V/U_\infty = 0.61$ (2900 rpm), and $Re_L = 1{,}030{,}000$.

Reynolds number is increased. High-speed motion pictures suggest the presence of helicoidal disturbances along the vortex lines just prior to the onset of turbulence. Helicoidal disturbances can be discerned as corkscrew deformations of the striations in Fig. 2.

When the striations appear toward the end of the midsection, as in Fig. 3, they are superimposed on the T-S waves which are similar to those present on the nonspinning body. This nonlinear superposition of the two instability modes raises interesting questions for the experts on nonlinear theory and for the computer predictors of transition. Would this pattern correspond to a still different topological breakdown to turbulence? Could this new type of transition be predicted by the e^n method? We hope to develop enough detailed hot-wire information in the next phase of our research program to make possible a quantitative test of predictive computer codes.

¶It has generally been assumed that the breakdown associated with the trusses is topologically equivalent to the breakdown of T-S waves documented quantitatively by Klebanoff, Tidstrom, and Sargent.[5] The assumption is supported by some of their hot-wire signatures, but no convincing comparison has been carried out.

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